Benha University
Faculty of Engineering- Shoubra
Electrical Engineering Department

Final Term Exam Date: 5-1-2015
Mathematics 3A Code : EMP271
Duration : 3 hours

- Answer all the following questions
- No. of questions: **8**
- The exam. Consists of **one** page
- Total marks: **80** [**10 marks each**]

1- Let X be a random variable with gamma distribution with alpha = 2, beta =1/5. Find the probability **P(X > 30), E(X) and Var(X).** **[Achieved ILOS**: **b1**]

2- Each of eight randomly selected tea drinkers is given a glass containing tea S and a glass containing tea F, the probability of choosing tea S is twice the probability of choosing tea F, let X is the number of individuals prefer tea F, what is the probability that at least 3 individuals choose tea S. **[Achieved ILOS**: **a1]**

3- Evaluate **m.g.f**. for the random variable of exponential and gamma distributions, then deduce **expected value and variance**. **[Achieved ILOS**: **a5, c1]**

4- Find **E(X), E(Y) , Cov(X,Y), & P(X > Y)** for the joint density function of two r.v's X and Y is given by $f(x,y)=\begin{cases} xy/96, & 0<x<4,\ 1<y<5 \\ 0 & \text{otherwise} \end{cases}$. **[Achieved ILOS**: **b2**]

5- Suppose we randomly select 5 balls from a urn contains 30 red balls and 20 black balls. Let the random variable is the number of red balls, what is the probability of selecting at least 3 red balls? **[Achieved ILOS: b7]**

6- f(x) = x, 0 < x < 1, expand in (i)**cosine harmonic** (ii)**odd harmonic** **[Achieved ILOS: c1]**

7- Find Fourier series for the function $f(x) = x^2$, $0 < x < 2$, in even sine harmonic and find $\sum_{n=1}^{\infty} \frac{1}{n^6}$. **[Achieved ILOS: c1]**

8- In a bolt factory, machines A,B,C manufacture such that machine A produce twice that of machine B which produce half that of machine C, 2%, 4%, 5% are defective bolts respectively , a bolt is drawn at random and it is a defective quality , what is the probability that it was produced by machine A, B, C. **[Achieved ILOS: a1, b1]**

Board of examiners:
Dr. eng. Khaled El Naggar

1- $P(X > 30) = \frac{1}{25}\int_{30}^{\infty} x\, e^{-x/5}\, dx$, put y = x-30, therefore

$$P(X > 30) = \frac{1}{25}\int_{0}^{\infty}(y+30)\, e^{-(y+30)/5}\, dy = \frac{e^{-6}}{25}\int_{0}^{\infty} y\, e^{-y/5}\, dy + \frac{6e^{-6}}{5}\int_{0}^{\infty} e^{-y/5}\, dy$$

Put y/5 = z $\Rightarrow$ dz = dy/5, therefore

$$P(X > 30) = e^{-6}\int_{0}^{\infty} z\, e^{-z}\, dz + 6e^{-6}\int_{0}^{\infty} e^{-z}\, dz = 7e^{-6}$$

$E(X) = \alpha / \beta = 10$, $Var(X) = 50$

2- P(S) = 2P(F), and P(S) + P(F) = 1, therefore P(S) = 2/3= q , P(F) = 1/3 = p, n = 8.

By binomial distribution, we get $P(X\le 5) = 1-P(X\ge 6) = 1- (P(X=6)+ P(X=7)+ P(X=8))$

$$= 1-\sum_{x=6}^{8} {}^{8}c_x (1/3)^x (2/3)^{8-x} = 1- 0.01966 = 0.98034$$

3- The moment generating function of a exponential distribution is expressed by $E(e^{tx}) = \int_{0}^{\infty} e^{tx}(\lambda\, e^{-\lambda x})dx = \int_{0}^{\infty} \lambda\, e^{-(\lambda - t)x}\, dx = \frac{\lambda}{(\lambda - t)}$, $\mu'_0 = 1$, $\mu'_1 = E(X) = \frac{1}{\lambda}$, $\mu'_2 = E(X^2) = \frac{2}{\lambda^2}$

The moment generating function of gamma distribution can be expressed by

$$E(e^{tx}) = \int_{0}^{\infty} e^{tx}\left(\frac{\beta^{\alpha}}{\Gamma\alpha} x^{\alpha-1} e^{-\beta x}\right)dx = \frac{\beta^{\alpha}}{\Gamma\alpha}\int_{0}^{\infty} x^{\alpha-1} e^{-(\beta - t)x} dx$$

Put $(\beta - t)x = y \Rightarrow dx = \frac{dy}{\beta - t}$, thus $E(e^{tx}) = \frac{\beta^{\alpha}}{(\beta - t)^{\alpha}\Gamma\alpha}\int_{0}^{\infty} y^{\alpha-1} e^{-y} dy = \frac{\beta^{\alpha}}{(\beta - t)^{\alpha}}$

4- The marginal probabilities $f_1(x)$, $f_2(y)$ are expressed by:

$f_1(x) = \int_1^5 \frac{xy}{96}\,dy = \frac{xy^2}{192}\Big|_1^5 = \frac{x}{8}$ and $f_2(y) = \int_0^4 \frac{xy}{96}\,dx = \frac{x^2y}{192}\Big|_0^4 = \frac{y}{12}$, therefore they are independent and $E(X) = \int_0^4 \frac{x^2}{8}\,dx = \frac{x^3}{24}\Big|_0^4 = \frac{8}{3}$, $E(Y) = \int_1^5 \frac{y^2}{12}\,dx = \frac{y^3}{36}\Big|_1^5 = \frac{31}{9}$, but $E(XY) = E(X)E(Y) = \frac{248}{27}$ and $E(2X + 3Y) = 2E(X) + 3E(Y) = \frac{16}{3} + \frac{31}{3} = \frac{47}{3}$

5- N= 50, k = 30, n =5, $p(x \geq 3) = \sum_{x=3}^{5} [\,{}^{k}C_x\,]\,[\,{}^{N-k}C_{n-x}\,] / [\,{}^{N}C_n\,]$

6-i) we have to extend this function to be even such that:

$$a_0 = \frac{2}{1}\int_0^1 x\,dx = (\frac{2x^2}{2})_0^1 = 1$$

$$a_n = \frac{2}{1}\int_0^1 x\ \cos(\frac{n\pi x}{1})dx = 2[x\frac{\sin(n\pi x)}{n\pi} + \frac{\cos(n\pi x)}{n^2\pi^2}]_0^1 = 2[\frac{\cos(n\pi) - 1}{n^2\pi^2}]$$

Therefore $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\frac{n\pi x}{T}) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{-4}{n^2\pi^2}\cos(2n\pi x)$,

ii) Thus $f(x) = \sum_{n=1}^{\infty} a_{2n-1}\cos(2n-1)x + \sum_{n=1}^{\infty} b_{2n-1}\sin(2n-1)x$

$$a_{2n-1} = \frac{2}{1}\int_0^1 x\cos(2n-1)\pi x\,dx$$

$$= \frac{2}{1}(x(\frac{\sin(2n-1)\pi x}{(2n-1)\pi}) - (\frac{-\cos(2n-1)\pi x}{(2n-1)^2\pi^2}))_0^1 = \frac{-4}{(2n-1)^2\pi^2}$$

$$b_{2n-1} = \frac{2}{1}\int_0^1 x \ \sin(2n-1)\pi x \, dx$$

$$= \frac{2}{1}\left(x\left(\frac{-\cos(2n-1)\pi x}{(2n-1)\pi}\right) - \left(\frac{-\sin(2n-1)\pi x}{(2n-1)^2\pi^2}\right)\right)_0^1$$

$$= \frac{2}{\pi(2n-1)}$$

Therefore f(x) = $\sum_{n=1}^{\infty} b_{2n-1} \sin(2n-1)x = -\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos n\pi}{(2n-1)^2}\sin(2n-1)x$

7- $f(x) = \sum_{n=1}^{\infty} b_{2n} \sin(\frac{2n\pi x}{T})$, where $a_0 = a_{2n} = 0$, and

$$b_{2n} = \frac{4}{T}\int_0^{T/2} f(x)\sin(\frac{2n\pi x}{T})\,dx = \frac{4}{4}\int_0^2 x^2 \sin(\frac{n\pi x}{2})\,dx$$

$$= \left(x^2\left(-\frac{2\cos(\frac{n\pi x}{2})}{n\pi}\right) - 2x\left(-\frac{4\sin(\frac{n\pi x}{2})}{n^2\pi^2}\right) + 2\left(\frac{8\cos(\frac{n\pi x}{2})}{n^3\pi^3}\right)\right)_0^2$$

$$= \frac{-8}{n\pi}\cos n\pi + \frac{16(\cos(n\pi)-1)}{n^3\pi^3}$$

8- Let the defective event is D and the probability of machines A,B,C are 2/5, 1/5, 2/5respectively, also P(D/A) = 0.02, P(D/B) = 0.04, P(D/C) = 0.05, therefore P(C/D) = $\frac{P(D/C)P(C)}{P(D)}$, P(B/D) = $\frac{P(D/B)P(B)}{P(D)}$, P(D) = P(D/A)P(A) + P(D/B)P(B) + P(D/C)P(C)